Resolving lithological units in the vadose zone from temporal changes in electrical conductivity

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H31E-0666

Stanford EARTH

I. Abstract

- We propose to resolve lithologic variation in the vadose zone by examining temporal changes in electrical conductivity
- Under certain assumptions, changes in electrical conductivity can directly indicate temporal changes in water content
- Use of the continuous wavelet transform (CWT) differentiation method on synthetic data shows that different lithologies have different rates of change in water content over time: clays do not change significantly, silts change moderately, and sands change significantly, particularly near the onset of infiltration
- Application of the CWT differentiation method to electrical conductivity probe data from a region of known lithology supports these observations

II. Background and motivation

- Obtaining estimates of hydraulic properties in the vadose zone is a challenge
  - ➢ Laboratory experiments using core samples from the field are not typically representative of true field conditions
  - ➢ Direct hydrologic field measurements are typically limited in spatial extent
- We use electrical conductivity data to examine these hydraulic properties (and hence lithologies) because they are:
  - ➢ Highly-correlated with soil water content
  - ➢ Directly sensitive to field conditions
  - ➢ Measured over a large spatial extent and over a long period of time
- In particular, we examine the rate of change in electrical conductivity over time to distinguish between different lithologies

III. Theory

A. Connecting lithology to temporal changes in water content

- Richards equation (Richards, 1931) is the governing hydrological process model that relates lithology to temporal changes in water content for unsaturated, non-steady state fluid flow:

$$\frac{\partial\theta(h)}{\partial t} = \frac{\partial\theta}{\partial h}\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z}+1\right)\right] + \frac{\partial}{\partial x}\left[K(h)\frac{\partial h}{\partial x}\right]$$

h [m] : matric head

$K(h)$ [m/s] : unsaturated hydraulic conductivity

$\theta(h)$ [–] : water retention

- Hydraulic properties $K(h)$ and $\theta(h)$ are non-linear functions of matric head h, but they can be parameterized by the van Genuchten-Mualem (VGM) model (Mualem, 1976; van Genuchten, 1980):

$$K(h) = K_s S_e(h)^{1/2}\left[1-\left(1-S_e(h)^{1/m}\right)^m\right]^2$$

where the effective saturation S_e is given by:

$$S_e(h) = \frac{\theta(h)-\theta_r}{\theta_s-\theta_r} = (1+|\alpha h|^n)^{-m}$$

$$m = 1-1/n$$

VGM parameters

θ_s [–] : saturated water content

θ_r [–] : residual water content

K_s [m/s] : saturated hydraulic conductivity

α [1/m] : related to air entry value

n [–] : related to pore size distribution

B. Connecting temporal changes in water content to temporal changes in electrical conductivity

- Water content is often related to electrical conductivity through Archie's law (Archie, 1942):

$$\sigma = \sigma_w\phi^m S_w^n = \sigma_w\phi^{m-n}\theta^n$$

σ [S/m] : electrical conductivity

σ_w [S/m] : electrical conductivity of pore fluid

n [–] : saturation exponent

m [–] : cementation exponent

ϕ [–] : porosity

θ [–] : water content

S_w [–] : water saturation (θ/ϕ)

- We cannot directly determine water content (and hence lithology type) from electrical conductivity measurements
- However, we can interpret temporal changes in electrical conductivity as a direct indicator of temporal changes in water content:

$$\boxed{\frac{d\sigma}{dt} = \frac{d}{dt}\left(\sigma_w\phi^{m-n}\theta^n\right) \approx \sigma_w\phi^{m-n}\frac{d\theta^n}{dt}}$$

Assumptions:
parameters such as porosity, saturation exponent, cementation exponent, and electrical conductivity of the pore fluid do not vary significantly over time

- Thus, we expect that by calculating the time derivative of electrical conductivity time series, we can distinguish between different lithology types (via water content)

IV. Developing a method to calculate the rate of temporal change in water content

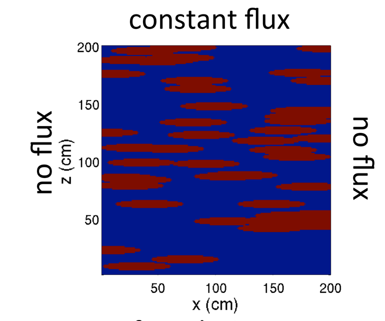
A. Forward modeling of water content

- We use HYDRUS-2D to solve Richards equation for time-varying water content distribution for various vadose zone models

Modeling Parameters

Geometry

- We examine five 2D models generated by SGeMS (example right)
- Each model is composed of a homogeneous background with elliptical anomalies randomly covering 30% of the domain
- Aim is to eliminate possible bias introduced by certain anomaly distributions



Soil Type	Sand	Silt	Clay
θ_r [–]	0.045	0.034	0.068
θ_s [–]	0.430	0.460	0.380
K_s [cm/day]	712.8	6.00	4.80
α [1/cm]	0.145	0.016	0.008
n [-]	2.68	1.37	1.09

(Carsel and Parrish, 1988)

Soil type

- Each lithology type has its own set of VGM parameters (table left)
- Four combinations of soil types used:
  - ➢ Clay anomalies in a sand background
  - ➢ Sand anomalies in a clay background
  - ➢ Silt anomalies in a sand background
  - ➢ Sand anomalies in a silt background
- Aim is to eliminate possible bias introduced by certain background lithology types

Infiltration scenario

Initial conditions: constant pressure head (set to 10% water saturation in sand) → Drainage 20 days → Onset of infiltration (light or heavy) → Infiltration 5 days → Drainage 15 days

B. Calculating the time derivative of a noisy signal

- Due to noise inherent in most geophysical data, we use the continuous wavelet transform (CWT) to estimate time derivatives (eg. Nie et al., 2002; Shao and Ma, 2003)
- The CWT of a signal $f(t)$ is given by:

$$Wf(a,b) = \int_{-\infty}^{\infty} f(t)\Psi^*_{a,b}(t)\,dt, \text{ where } \Psi^*_{a,b}(t) = \frac{1}{\sqrt{a}}\,\Psi\left(\frac{t-b}{a}\right)$$

$\Psi^*_{a,b}(t)$: mother wavelet

$*$: complex conjugate

a : dilation parameter

b : translation parameter

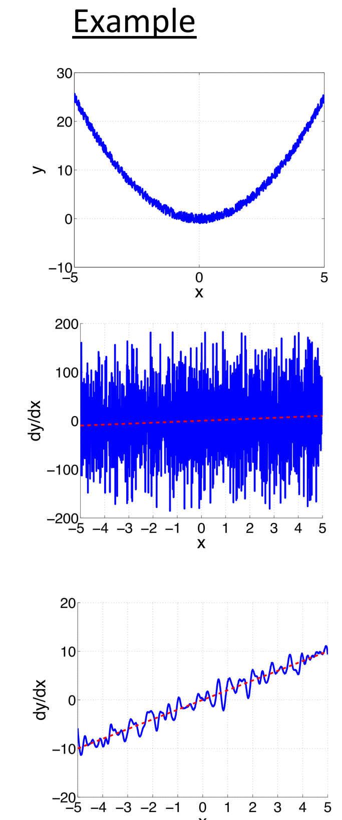
- If we take the wavelet to be the derivative of a smoothing function $\theta(t)$, we can show that:

$$Wf(a,b) = f(b)\otimes\Psi^*_a(b) = \left[\frac{d}{db}f(b)\right]\otimes\left[(-a)\theta^*_a(b)\right],$$

where $\Psi^*_a(b) = 1/\sqrt{a}\;\Psi^*(-b/a)$, $\theta^*_a(b) = \theta^*(-b/a)/\sqrt{a}$; $\otimes$: convolution

- The CWT differentiation method returns the derivative of a signal that has been smoothed by the smoothing function $\theta(t)$

Example



Noisy parabola

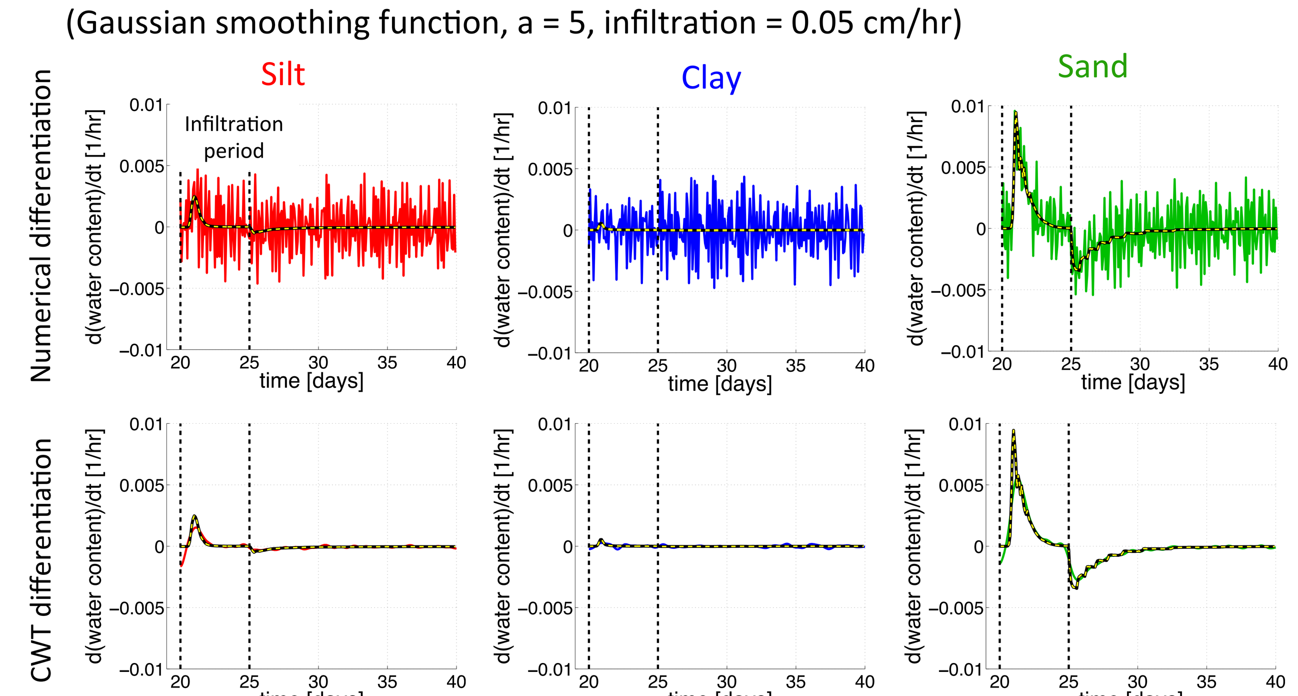
$y = x^2 + \mathcal{U}(-1,1)$

Numerical differentiation

CWT differentiation (a = 5)

C. Application of CWT differentiation to water content

- We differentiate synthetic noisy time series of water content in each lithology type at a point 20 cm deep (Gaussian smoothing function, a = 5, infiltration = 0.05 cm/hr)

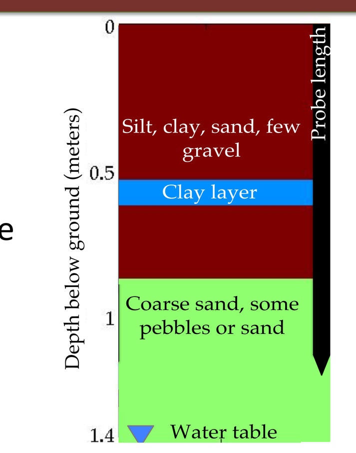


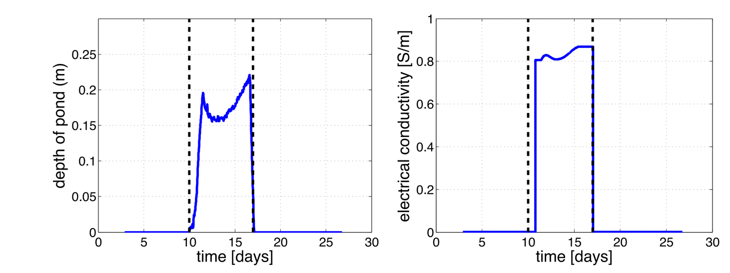
- The CWT method better matches the derivative of the pure signal (indicated by black and yellow lines)
- Each lithology type displays a distinct pattern, particularly at the onset of infiltration

V. Application to electrical conductivity field data

A. Field site and data

- Electrical conductivity probe data
- Collected at aquifer recharge and recovery project near Denver, CO
- Data recorded at 24-minute intervals
- 40 different electrode configurations in Wenner and Dipole-Dipole arrangements
- Geoprobe cuttings provided (right)

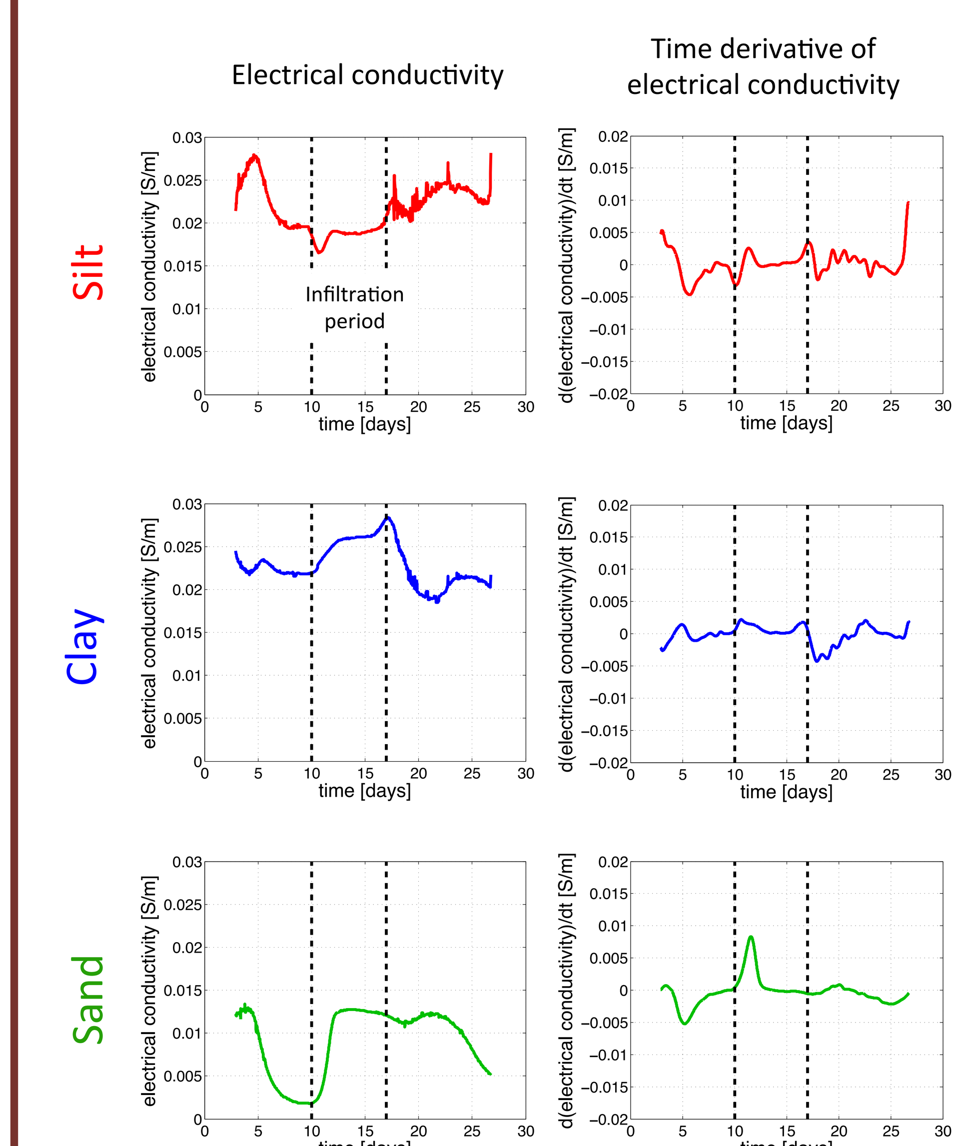




- Level of pond water (far left) reveals when to expect onsets of infiltration and drainage (vertical black dashed lines)
- Electrical conductivity of pond water does not change significantly over time (near left)

B. Results

- We calculate the rate of change in electrical conductivity (and hence water content) over time at depths corresponding to expected silt, clay, and sand using CWT differentiation



- Time derivatives of electrical conductivity resemble those of water content calculated in our synthetic study (Section IV.C), particularly at the onset of infiltration
- The time derivatives of electrical conductivity in both clay and silt show relatively little variation
- Silt shows slightly more variation than clay, most noticeably around the onset of infiltration
- Sand is the easiest lithology type to distinguish
- The time derivative of its electrical conductivity displays the strongest peak at the onset of infiltration
- Both silt and clay are easily distinguishable from sand using this method

VI. Conclusions

- We can distinguish between different lithology types in the vadose zone by examining the rate of change in electrical conductivity over time
- The time derivatives of both synthetic water content and electrical conductivity probe data show that changes are most easily observed near the onset of infiltration
- Sand is the easiest lithology type to distinguish because it displays the strongest peak at the onset of infiltration
- Although both silt and clay display relatively little variation, silt displays a slightly stronger peak than clay at the onset of infiltration

Acknowledgements

We would like to thank Chloe Mawer, Adam Pidlisecky, Denys Grombacher, Dave Cameron, Stewart Levin, and Stephen Moysey for helpful suggestions and guidance. We would like to especially thank Chloe Mawer for access to the probe data.

References

Archie, G. E., 1942, The electrical resistivity log as an aid in determining some reservoir characteristics: I. Pet Tech, **5.**
Carsel, R.F., and Parrish, R. S., 1988, Developing joint probability distributions of soil water retention characteristics: Water Resources Research, **24**, no. 5, 755—769.
Mualem, Y., 1976, A new model for predicting the hydraulic conductivity of unsaturated porous media: Water Resources Research, **12**, no. 3, 513—522.
Nie, L., Wu, S., Lin, X., and Rui, L., 2002, Approximate derivative calculated by using continuous wavelet transform: Journal of Chemical Information and Computer Sciences, **42**, no. 2, 274—283.
Richards, L. A., 1931. Capillary conduction of liquids through porous mediums: Physics, **1**, no. 5, 318—333.
Shao, X., and Ma, C., 2003, A general approach to derivative calculation using wavelet transform: Chemometrics and Intelligent Laboratory Systems, **69**, no. 1, 157—165.
van Genuchten, M. T., 1980, A closed-form equation for predicting the hydraulic conductivity of unsaturated soils: Soil Science Society of American Journal, **44**, no. 5, 892—898.